TEST-final–exam – Phys 112.1 (Thermal Physics)
Nov 29, 2000

1.) Determine the density of orbitals $D(\epsilon)$ for a harmonic oscillator in one dimension. (20 points)

2.) a) For relativistic (nearly mass less) Fermions the energy ϵ and momentum p are related by $\epsilon = c|p|$ where c is the velocity of light. The one–particle partition function in a *dilute* gas is given by

$$Z_1 = \frac{V}{h^3} \int_{-\infty}^{+\infty} dp_x \int_{-\infty}^{+\infty} dp_y \int_{-\infty}^{+\infty} dp_z \, exp(-\frac{\epsilon}{\tau}) \tag{1}$$

Evaluate Z_1 as function of V and τ. (Useful integral $\int_{-\infty}^{+\infty} dy\, y^l\, e^{-y} = l!$) (20 points)

b) Find the free energy $F = -\tau \log Z_N$ and evaluate the energy density U/V as a function of τ. Give the energy per particle as a function of τ. (20 points)

c) Give an expression for the entropy (analogous to the Sakur Tetrode law for nonrelativistic particles). (20 points)

3) Consider a lattice of fixed hydrogen atoms; suppose that each atom can exist in four states:

State	Number of electrons	Energy
Ground	1	$-\frac{1}{2}\Delta$
Positive Ion	0	$-\frac{1}{2}\delta$
Negative Ion	2	$\frac{1}{2}\delta$
Excited	1	$\frac{1}{2}\Delta$

a) Calculate the Gibbs sum for this system. (10 points)

b) Calculate the average number of electrons per atom. (20 points)

c) Find the condition that the average number of electrons per atom be unity. (20 points)

4) Ascent of Sap in Trees
Find the maximum height to which water may rise in a tree under the assumption that the roots stand in a pool of water and the uppermost leaves are in air containing water vapor at a relative humidity $r = 0.9$. The temperature is 27°C. (If the relative humidity is r, the actual concentration is rn_0, where n_0 is the concentration in the saturated air that stands immediately above the pool of water.) (40 points)
$m_{H_2O} = 3 \cdot 10^{-23}\, g;\ g = 10\, m/sec^2$

5) a)Show that the average pressure in a system in thermal contact with a heat reservoir is given by

$$p = -\frac{\sum_s (\partial\epsilon_s/\partial V)_N exp(-\epsilon_s/\tau)}{Z} \tag{2}$$

where the sum is over all states of the system. (20 points)
b) Show for a gas of free particles that

$$\left(\frac{\partial\epsilon_s}{\partial V}\right)_N = -\frac{2}{3}\frac{\epsilon_s}{V} \tag{3}$$

as a result of the boundary conditions of the problem. (20 points)
c) Show, that for a gas of free nonrelativistic particles

$$p = \frac{2U}{3V} \tag{4}$$

where U is the thermal average energy of the system. (20 points)

6) In relativistic heavy ion collisions, nucleons emerge at a temperature of $\tau = 170\, MeV$ and a density of $n_0 = 0.16\, fermi^{-3}$ ($1\, fermi = 10^{-13}\, cm$). Show that, even though the nucleons are fermions, it is justified to treat them in the classical limit under these conditions. (30 points)
$m_n c^2 = 940\, MeV;\ \hbar c = 197 MeV\, fermi$
BONUS of 20 points: Calculate the chemical potential μ of the nucleons under these conditions.